2.5 Determinants & Multiplicative Inverses of Matrices

Each square matrix has a determinant.

Second-order determinant= the determinant associated with a 2 x 2 matrix.

- determinant for a 2 x 2 matrix is a number

Ex 1 Find the value of $\begin{vmatrix} 0 & -2 \\ 8 & -6 \end{vmatrix} = 0 - {}^{-}16 = 16$

-16, 0

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matrix with a non-zero determinant.

(determinant for a 3 x 3 matrix)

Rewrite the first 2 columns.

Ex 2 Find the value of $\begin{vmatrix} -4 & -6 & 2 \\ 5 & -1 & 3 \\ -2 & 4 & -3 \end{vmatrix} \begin{matrix} -4 & -6 \\ 5 & -1 \\ -2 & 4 \end{matrix}$

$4 - 48 + 90 = 46$

$-12 + 36 + 40 = 64$

bottom - top

$64 - 46 = 18$

Identity Matrix for Multiplication: a square matrix, **I**, so that

$I \times A = A$ A = any matrix

$\begin{bmatrix} & \\ & \end{bmatrix}$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Some square matrices have a multiplicative inverse

A^{-1} = inverse matrix of A

The product of a matrix and its inverse is a square identity matrix.

*** Not all matrices have an inverse***

$A \cdot A^{-1} = I$

matrix · inverse of the matrix = Identity Matrix

Inverse of a Second Order Matrix

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$, then $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

inverse of matrix A.

$(\det A) \neq 0$

*** If | A | = 0, then A^{-1} does not exist and the matrix is Singular.

If $\det A = 0$, then the inverse of A

Ex 3 Find the inverse matrix for $\begin{bmatrix} 8 & 9 \\ 3 & -1 \end{bmatrix}$.

$\det A = \begin{vmatrix} 8 & 9 \\ 3 & -1 \end{vmatrix} = -8 - 27 = -35$

27, -8

$A^{-1} = \frac{1}{-35}\begin{bmatrix} -1 & -9 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} \frac{1}{35} & \frac{9}{35} \\ \frac{3}{35} & \frac{-8}{35} \end{bmatrix} = A^{-1}$

Inverse matrices can be used to solve systems of equations.

A = the coefficient matrix (just the coefficients of the variables)
X = the variables (a column matrix of just the variables)
B = the constant matrix (a column matrix of just the constants)

Since A * X = B we can use $X = A^{-1} * B$ to solve the system.

inverse of matrix A × matrix B

(This only works if A^{-1} exists) - if $|A| \neq 0$

det

Ex 4 Use inverse matrices to solve the system:

$4x - 2y = 16$
$x + 6y = 17$

$A = \begin{bmatrix} 4 & -2 \\ 1 & 6 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \end{bmatrix}$ $B = \begin{bmatrix} 16 \\ 17 \end{bmatrix}$

A^{-1} ① $\det A = \begin{vmatrix} 4 & -2 \\ 1 & 6 \end{vmatrix} = 24 - {}^{-}2 = 26 \neq 0$

-2, 24

② $\frac{1}{26}\begin{bmatrix} 6 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{13} & \frac{1}{13} \\ \frac{-1}{26} & \frac{2}{13} \end{bmatrix} = A^{-1}$

$A^{-1} \cdot B = X$

$\begin{bmatrix} \frac{3}{13} & \frac{1}{13} \\ \frac{-1}{26} & \frac{2}{13} \end{bmatrix} \cdot \begin{bmatrix} 16 \\ 17 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ $x = 5$, $y = 2$